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DROPLET DEPOSITION IN AN ANNULAR GEOMETRY

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Abstract—An analytical approach to the problem of droplet deposition in annular flow through an annulus is presented. The underlying theory, due to Hutchinson *et al.* (1971), allows the droplet deposition to be treated as a diffusion process and the results obtained enable some properties of mass transfer coefficients in non-circular geometries to be explained.

1. INTRODUCTION

Annular flow is one of the most important two-phase flow regimes as it is found in many types of industrial equipment, for example water-cooled nuclear reactors and once through serpentine boilers. In the annular flow regime the liquid phase flows as a thin film along the wall (or walls) of the equipment and as droplets through the gas phase (figure 1). The liquid droplets are entrained from and subsequently deposited onto the liquid film. Consequently any mathematical model of annular two-phase flow must describe the processes of liquid droplet entrainment and deposition, and this paper is concerned with some aspects of the latter.

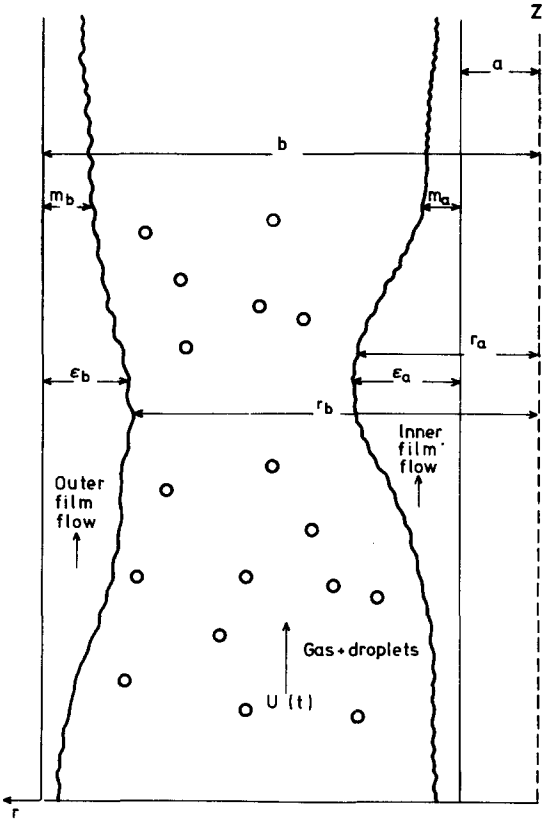

Figure 1. The annular geometry and coordinate system.

There are essentially two descriptions of droplet deposition which have been incorporated into mathematical models of annular two-phase flow. The first of these assumes that droplet deposition is a mass transfer process in which the flux of droplets onto the film is proportional to the mean concentration of droplets in the gas core. The "constant" of proportionality, called the mass transfer coefficient, has been shown experimentally to be fairly insensitive to the flow rates of gas and liquid for a given system pressure in a round tube geometry (Cousins *et al.* 1965; Cousins & Hewitt 1968). In addition a model of heated annular flow, due to Whalley *et al.* (1974), which incorporates the above description of droplet deposition gives acceptable predictions of the liquid flow distribution. The weak link in this approach is that the mass transfer coefficient must be known *a priori* and at present is chosen by reference to a correlation developed by Whalley *et al.* (1974). The correlation relates the mass transfer coefficient to the surface tension and has little, if any, theoretical basis.

The second description of deposition is due to Hutchinson *et al.* (1971) and assumes that the droplets in the gas core interact with the turbulent eddies in such a way that the deposition process is essentially diffusive. The predictions of this theory agree well with experimental data and at first sight the second approach would appear preferable to the first in that it is more soundly based. However it too suffers from a weak link in that it requires specification of a diffusion coefficient. A calculation procedure for the diffusion coefficient is detailed in Hutchinson *et al.* (1971) but requires the mean value of the droplet size, which is also unknown *a priori.*

As a result the usual approach to droplet deposition taken in mathematical models is the simpler mass transfer method, although Hutchinson *et al.* (1974) have shown that the diffusion approach can lead to slightly improved results.

The above discussion relates to annular flow in a round tube, but many of the geometries in which annular two-phase flow occurs are far more complicated. In order to extend to more complicated geometries any model, such as that of Whalley *et al.* (1974), which has been developed for a round tube geometry, great care must be taken to determine which of the various assumptions peculiar to a round tube have more general validity. In annular two-phase flow correlations and assumptions which relate specifically to the film, for example that of constant shear stress throughout the film, may be expected to be applicable in any geometry for which the films are thin and well separated. However, those empirical relationships which also relate to the gas core, in particular the expressions for the deposition and entrainment mass fluxes, require more careful analysis.

The problem reduces essentially to that of specifying the mass transfer coefficients for deposition onto (and entrainment from) each film covered surface.

Whalley *et al.* (1975) have extended the heated annular flow model mentioned above to an annular geometry by splitting the flow area into two regions and using the concept of a hydraulic diameter. The mass transfer coefficients for the inner and outer surfaces were determined by optimizing the agreement between the predictions of the model and available experimental data. It was found that the mass transfer coefficients were of the same order as their values for a round tube at the same pressure and that the predictions of the model were not particularly sensitive to the way in which the flow area was divided. These findings enabled Whalley (1976) to further extend the model to a rod-bundle geometry by assuming that the mass transfer coefficients for each (rod-centred) subchannel were equal and that the method of subdivision into subchannels could be based on geometry alone.

It is the purpose of this paper to deduce, from the assumption that the deposition process is described by the model of Hutchinson *et al.* (1971), relationships between the mass transfer coefficients and diffusion coefficient for droplet deposition in an annular geometry. Qualitative conclusions concerning the specification of mass transfer coefficients and the method of division into subchannels for more complicated geometries can then be drawn.

2. MATHEMATICAL FORMULATION

2.1 *Derivation of the diffusion equation*

The process by which particles (solid or liquid) flowing in a turbulent gas stream are deposited in a circular tube is modelled in two stages by Hutchinson *et al.* (1971). In the first stage the fraction of particles which approach the wall is calculated by assuming that the radial drag on a particle due to the motion of turbulent eddies in the gas results in the particle executing a random walk in the cross-section of the tube. The particle concentration profile then satisfies a diffusion equation in which the diffusion coefficient is the mean square particle displacement per unit time. This coefficient can then be evaluated from a consideration of the interaction between the large, energy containing eddies and the particle and, for a round tube, may be taken to be constant as a first approximation. For flow through an annulus it is by no means obvious that the diffusion coefficient should be radially independent of position. However, provided that there is not a large degree of asymmetry in the distribution of length scales of the energy containing eddies in a cross-section of the annulus it is reasonable to expect the mean square particle displacement due to eddy interaction, and hence the diffusion coefficient, to be independent of radial position.† In the second stage of the model, the fraction of those particles which approach the wall that are actually deposited is calculated. This fraction depends on the thickness of the viscous gas sublayer, but for the purposes of droplet deposition in annular two-phase flow may be taken to be unity.

The diffusion equation for the annulus is best obtained by formulating the problem in cylindrical polar coordinates (r, θ, z), as shown in figure 1, in which the annular gap $a \leqslant r \leqslant b$ is occupied by liquid droplets of concentration $C(r, z)$, axial symmetry being assumed. A mass balance over a fixed elementary volume gives

$$\int_V \left\{\frac{\partial}{\partial z}[U(z)C(r, z)] - S(r, z)\right\} \mathrm{d}V = \int_\Sigma \lambda \nabla_r C(r, z) \cdot \mathbf{n}\, \mathrm{d}\Sigma, \qquad [2.1]$$

where $U(z)$ is the mean gas core velocity, with which the droplets are assumed to move, λ is the diffusion coefficient, V and Σ denote, respectively, the elementary volume and its surface, $\mathbf{n}$ is the unit vector normal to the surface,

$$\nabla_r = \hat{\mathbf{r}} \frac{\partial}{\partial r}, \qquad [2.2]$$

$\hat{\mathbf{r}}$ being a unit normal vector in the radial direction, and $S(r, z)$ is a source term representing the droplet production due to entrainment. Equation [2.1] assumes that steady state conditions $(\partial/\partial\tau \equiv 0)$ have been achieved and give, on applying Green's theorem,

$$\frac{\partial}{\partial z}\{U(z)C(r, z)\} - S(r, z) = \lambda \nabla_r^2 C(r, z), \qquad [2.3]$$

where

$$\nabla_r^2 \equiv \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}. \qquad [2.4]$$

†Of course, the diffusion coefficient could depend on downstream position if the flow is developing, but it is only under conditions where the axial velocity varies rapidly that we would expect axial variations in the diffusion coefficient to be significant. In any event, the asymptotic results to be presented in the paper are still valid.

Finally, if we transform to a system of coordinates which move with the gas velocity, i.e. if we write

$$z = z_0 + \int_{t_0}^{t} U(t')\,dt', \tag{2.5}$$

so that

$$\frac{\partial}{\partial z} \equiv \frac{1}{U(t)}\frac{\partial}{\partial t}, \tag{2.6}$$

we obtain

$$\frac{\partial}{\partial t}\{U(t)C(r,t)\} - U(t)S(r,t) = \lambda \nabla_r^2\{U(t)C(r,t)\}. \tag{2.7}$$

Equation [2.7] is an evolution equation for the concentration profile $C(r, t)$ and is clearly a diffusion equation. It is assumed that the liquid films at the inner and outer walls of the annulus are perfect absorbers of the droplets and, furthermore, that as far as the gas core is concerned, the films are infinitely thin. The boundary conditions for [2.7] are therefore

$$C(a,t) = C(b,t) = 0. \tag{2.8}$$

At the inlet to the tube, $z = z_0$ (i.e. $t = t_0$), we assume that the droplets are distributed evenly over the annular region $r_1 \leqslant r \leqslant r_2$, ($a < r_1$, $r_2 < b$). The initial conditions for [2.7] may then be written

$$C(r,0) = C_p \frac{b^2 - a^2}{r_2^2 - r_1^2} H(r_2 - r)H(r - r_1), \tag{2.9}$$

where $H(X)$ denotes the step function, defined by

$$H(X) = 0 \quad \text{if} \quad X < 0; \qquad H(X) = 1 \quad \text{if} \quad X > 0, \tag{2.10}$$

and C_p is a superficial concentration defined with reference to the complete annulus cross-section.

2.2 *The source term*

The source of droplets is assumed to be at the tips of roll-waves, which are known to occur on thin liquid films in annular two-phase flow (Hewitt & Hall-Taylor 1970). If we assume that such a source can be replaced by continuous ring sources at $r_a(t)$ and $r_b(t)$, of strength $E_a(t)$ and $E_b(t)$ respectively, then the source term may be written as

$$S(r,t) = \frac{a}{r_a(t)} E_a(t)\delta\{r - r_a(t)\} + \frac{b}{r_b(t)} E_b(t)\delta\{r - r_b(t)\}, \tag{2.11}$$

where $\delta(r)$ is the Dirac delta function. The factors $a/r_a(t)$ and $b/r_b(t)$ appear in [2.11] because $E_a(t)$ and $E_b(t)$ are superficial mass fluxes defined per unit area of inner and outer tube wall, respectively.

2.3 *Solution of the equation*

The solution of [2.7] is quite straightforward and may be written

$$C(r, t) = \frac{1}{U(t)}[T_a(r, t) + T_b(r, t) + T_p(r, t)], \qquad [2.12]$$

where T_a, T_b and T_p represent the contributions to the solution from the sources at $r_a(t)$ and $r_b(t)$ and the plug source (corresponding to the inlet flow) at $t = 0$. They are given by

$$T_s(r, t) = \frac{\pi^2 s}{2} \sum_{n \geq 0} \frac{\alpha_n^2 J_0^2(a\alpha_n) U_0(\alpha_n r)}{J_0^2(\alpha_n a) - J_0^2(\alpha_n b)} \int_0^t e^{-(t-t')\lambda\alpha_n^2} \{U(t') E_s(t') U_0[\alpha_n r_s(t')]\} \, dt', \qquad [2.13]$$

where s stands for either a or b and

$$T_p(r, t) = \frac{\pi^2 C_p U(0)}{2} \cdot \frac{b^2 - a^2}{r_2^2 - r_1^2} \cdot \sum_{n \geq 0} \frac{J_0^2(\alpha_n a) U_0(\alpha_n r)}{J_0^2(\alpha_n a) - J_0^2(\alpha_n b)} I(r_2, r_1) \, e^{-\lambda\alpha_n^2 t}. \qquad [2.14]$$

In [2.13] and [2.14] $U_0(\alpha r)$ is defined by

$$U_0(\alpha r) = J_0(\alpha r) Y_0(\alpha b) - Y_0(\alpha r) J_0(\alpha b). \qquad [2.15]$$

J_0 and Y_0 are the Bessel functions of the first kind of zero order, α_n is the nth root of $U_0(\alpha_n a) = 0$, the summation is taken over the positive roots α_n and, finally,

$$I(r_2, r_1) = \left[r \frac{dU_0(\alpha_n r)}{dr} \right]_{r_1}^{r_2}. \qquad [2.16]$$

2.4 *Asymptotic solutions*

We are primarily interested in the solution at large distances from the inlet, when equilibrium has been achieved, and therefore require the asymptotic form of solution [2.12] as $t \to \infty$. Making use of the result that, for reasonably well-behaved functions $\chi(t)$ which tend to a non-zero limit $\bar{\chi}$ as $t \to \infty$,

$$\lim_{t \to \infty} \int_0^t e^{-\lambda\alpha_n^2 (t-t')} \chi(t') \, dt' = \frac{\bar{\chi}}{\lambda\alpha_n^2}, \qquad [2.17]$$

we find that

$$\begin{aligned} \bar{C}(r) &= \lim_{t \to \infty} C(r, t) \\ &= \frac{a\bar{E}_a \pi^2}{2\lambda} \sum_{n \geq 0} \left[\frac{J_0^2(\alpha_n a) U_0(\alpha_n r) U_0(\alpha_n \bar{r}_a)}{J_0^2(\alpha_n a) - J_0^2(\alpha_n b)} \right] \\ &\quad + \frac{b\bar{E}_b \pi^2}{2\lambda} \sum_{n \geq 0} \left[\frac{J_0^2(\alpha_n a) U_0(\alpha_n r) U_0(\alpha_n \bar{r}_b)}{J_0^2(\alpha_n a) - J_0^2(\alpha_n b)} \right]. \end{aligned} \qquad [2.18]$$

The asymptotic deposition mass fluxes $\bar{D}_a$ and $\bar{D}_b$ are given by

$$\bar{D}_a = \lambda \frac{d\bar{C}}{dr}\bigg|_{r=a} \quad \text{and} \quad \bar{D}_b = -\lambda \frac{d\bar{C}}{dr}\bigg|_{r=b}, \qquad [2.19]$$

and may be written

$$\bar{D}_a = \frac{a\bar{E}_a \log \frac{b}{\bar{r}_a} + b\bar{E}_b \log \frac{b}{\bar{r}_b}}{a \log \frac{b}{a}}, \qquad [2.20]$$

$$\bar{D}_b = \frac{a\bar{E}_a \log \frac{\bar{r}_a}{a} + b\bar{E}_b \log \frac{\bar{r}_b}{a}}{b \log \frac{b}{a}}. \qquad [2.21]$$

As a check on the results so far we see from [2.20], [2.21] that

$$a\bar{E}_a + b\bar{E}_b = a\bar{D}_a + b\bar{D}_b, \qquad [2.22]$$

which confirms that overall mass conservation is satisfied.

It is appropriate to emphasize here that the problem has been formulated in a general way which allows not only for the fact that droplet deposition and entrainment are separate processes, governed by separate physical mechanisms, but also for the fact that deposition and entrainment could be entirely independent. This, of course, is not the case in annular two-phase flow. A result of the generality is [2.22] which simply expresses overall mass balance. However, in order to obtain solutions relevant to vertical annular two-phase flow in which the deposition and entrainment rates are asymptotically constant it is necessary to impose *separate* equilibrium at each surface. For if this is not the case consider what would happen if $\bar{E}_a > \bar{D}_a$. Under these conditions the film on the wall at $r = a$ would disappear at some axial location, thus making $\bar{E}_a$ zero and, as a consequence of the above inequality, $\bar{D}_a < 0$. This is not an acceptable result.† We therefore write

$$\bar{D}_a = \bar{E}_a, \qquad \bar{D}_b = \bar{E}_b, \qquad [2.23]$$

and with this added restriction [2.20], [2.21] become degenerate and give

$$\frac{\bar{E}_a}{\bar{E}_b} = \frac{\bar{D}_a}{\bar{D}_b} = \frac{b \log \frac{b}{\bar{r}_b}}{a \log \frac{\bar{r}_a}{a}}. \qquad [2.24]$$

2.5 *Average asymptotic concentration*

One final result is required before we can relate mass transfer coefficients to the diffusion coefficient, namely the expression for the average asymptotic concentration $\bar{C}_{av}$. Making use of the properties of the functions $U_0(\alpha_n r)$ we find

$$\begin{aligned}\bar{C}_{av} &= \frac{2}{b^2 - a^2}\int_a^b r\bar{C}(r)\,\mathrm{d}r \\ &= \left[a\bar{E}_a\left(b^2 \log \frac{\bar{r}_a}{a} + a^2 \log \frac{b}{\bar{r}_a} - \bar{r}_a^2 \log \frac{b}{a}\right)\right. \\ &\qquad \left. + b\bar{E}_b\left(b^2 \log \frac{\bar{r}_b}{a} + a^2 \log \frac{b}{\bar{r}_b} - \bar{r}_b^2 \log \frac{b}{a}\right)\right] \Big/ \left[2\lambda(b^2 - a^2) \log \frac{b}{a}\right].\end{aligned} \qquad [2.25]$$

†There is the theoretical possibility that the film thickness may appear and disappear intermittently in the downstream direction but this is not thought likely on physical grounds.

2.6 *Relationships between the mass transfer coefficients and the diffusion coefficient*

The mass transfer coefficients for deposition onto $r = a$ and $r = b$, k_a and k_b respectively, are defined by

$$k_a = \frac{\bar{D}_a}{\bar{C}_{av}}, \qquad k_b = \frac{\bar{D}_b}{\bar{C}_{av}}, \tag{2.26}$$

and use of [2.24] gives the following expression for the ratio of k_a to k_b:

$$\frac{k_a}{k_b} = \frac{b \log \frac{b}{\bar{r}_b}}{a \log \frac{\bar{r}_a}{a}}. \tag{2.27}$$

The mass transfer coefficients can also be related to the diffusion coefficient through [2.25] giving

$$k_a = \frac{\frac{2\lambda}{a}(b^2 - a^2) \log \frac{b}{\bar{r}_b}}{(b^2 - \bar{r}_b^2) \log \frac{\bar{r}_a}{a} - (\bar{r}_a^2 - a^2) \log \frac{b}{\bar{r}_b}}, \tag{2.28}$$

$$k_b = \frac{\frac{2\lambda}{b}(b^2 - a^2) \log \frac{\bar{r}_a}{a}}{(b^2 - \bar{r}_b^2) \log \frac{\bar{r}_a}{a} - (\bar{r}_a^2 - a^2) \log \frac{b}{\bar{r}_b}}. \tag{2.29}$$

As a check on the last two results we may take the limiting case a, $r_a \to 0$ and we find

$$k_b \to \frac{2\lambda b}{(b^2 - \bar{r}_b^2)}, \tag{2.30}$$

which agrees with the result obtained by Hutchinson *et al.* (1974) for a round tube.† Finally we derive approximations to the results given by [2.27]–[2.29] which will be useful for discussion purposes. For thin films it is known experimentally that the wave height on a film of thickness m is approximately $5m$ and therefore we may assume that $\bar{r}_a$ and $\bar{r}_b$ are close to a and b. We write

$$\bar{r}_a = a + \epsilon_a, \quad \epsilon_a \ll a; \qquad \bar{r}_b = b - \epsilon_b, \quad \epsilon_b \ll b, \tag{2.31}$$

and substituting into [2.27]–[2.29] we find, to second order in the ϵ terms, that

$$\frac{k_a}{k_b} = \frac{\epsilon_b}{\epsilon_a}\left(1 + \frac{\epsilon_a}{2a} + \frac{\epsilon_b}{2b}\right), \tag{2.32}$$

and, to first order in the ϵ terms

$$k_a \simeq \frac{\lambda}{\epsilon_a} \propto \frac{1}{m_a}; \qquad k_b \simeq \frac{\lambda}{\epsilon_b} \propto \frac{1}{m_b}. \tag{2.33}$$

†It is also found that $k_a \to \infty$ in the limit $\bar{r}_a, a \to 0$ which is what we could expect.

3. DISCUSSION

Before drawing any conclusions from the above analysis it is appropriate to comment on the simplicity of the results given by [2.33]. That the mass transfer coefficients are proportional to the diffusion coefficient is reasonable. But it might appear strange that the length scales ϵ_a and ϵ_b appearing in the denominator are characteristic of the film thickness, rather than the gas boundary layer thickness (as would occur, for example, in molecular diffusion). It is clear from the explanation given in section 2.1 why the thickness of the gas boundary layer plays no part in the present problem. Also, since ϵ is the distance between the droplet source and sink it is reasonable that the mass transfer coefficients should decrease as ϵ increases. The relation $\epsilon \simeq 5\ m$ is a consequence of the hydrodynamics of the film flow. Hence, if the model of droplet deposition proposed by Hutchinson *et al.* (1971) is acceptable, the results expressed by [2.33] are physically reasonable.

Unfortunately there appear to be no measurements of k_a, k_b, ϵ_a and ϵ_b for adiabatic flow in an annulus against which the above results can be tested. The work of Min *et al.* (1971) on adiabatic annular flow through an annulus, when analysed, predicts a mass transfer coefficient ratio of about 2–3. However, their results were obtained in a tube of length corresponding to 65 hydraulic diameters whereas an estimate of the distance required to attain equilibrium (see Hutchinson *et al.* 1974) shows that a tube of length of order 10^3 hydraulic diameters would be required before the asymptotic approximations made in section 2 are valid. We therefore appeal to data obtained in heated flow in order to draw support for the above results.

The result contained in [2.32] shows that, to first order, the ratio of the mass transfer coefficients is inversely proportional to the ratio of the film thicknesses. Whalley *et al.* (1975) found that the results of Jensen & Mannov (1974) for heated flow in an annulus could be accounted for satisfactorily when the dryout length was near a maximum for a given total heat flux, by taking the mass transfer coefficient ratio to be unity. The maximum dryout length occurs when the films on both surfaces of the annulus vanish at the same axial location. It is then plausible to assume that ϵ_a and ϵ_b are similar over a substantial length of the tube and the equality of the mass transfer coefficients is then predicted by [2.32]. In addition the predicted change in the mass transfer coefficient ratio with film thickness is, according to [2.32], small ($\sim 10\%$) when representative values are used for the variables. Such behaviour is consistent with Whalley *et al.* (1975) observation that even under conditions in which the dryout power, and hence the ratio of film thicknesses, varied, the use of a mass transfer coefficient ratio of unity still enables the experimental results to be predicted quite well. The point is illustrated in figure 2, which shows a comparison of the predictions of Whalley *et al.* (1975) with the results of Jensen & Mannov (1974) plotted in the form dryout quality vs fraction of electrical heating power applied to the outer tube. The continuous line corresponds to optimized values of the mass transfer coefficients. It is seen that not only is the optimized ratio of mass transfer coefficients quite close to one but also that the agreement between theoretical and experimental predictions of the quality at dryout does not deteriorate significantly by using a mass transfer ratio of one.

It should be noted that the experiments of Jensen & Mannov (1974) were carried out in a tube too short, again, for asymptotic approximations to be valid. It is, however, thought that Whalley *et al.* (1975) were able to predict their results with a mass transfer coefficient ratio in accord with the present theory for the following reason. Suppose that the approach to equilibrium occurs in a time of order τ^*. The length of tube required to attain equilibrium will then be of order $V^*\tau^*$, where V^* is a velocity typical of the gas core. In unheated flows, V^* will be fairly constant and relatively large whereas in a heated flow, for a significant time, the quality will be low and hence V^* will be relatively small. If it is legitimate to assume τ^* to be of the same order of magnitude in both heated and unheated flows, the above arguments lead to the conclusion that the length of tube required to reach equilibrium in unheated flow will be

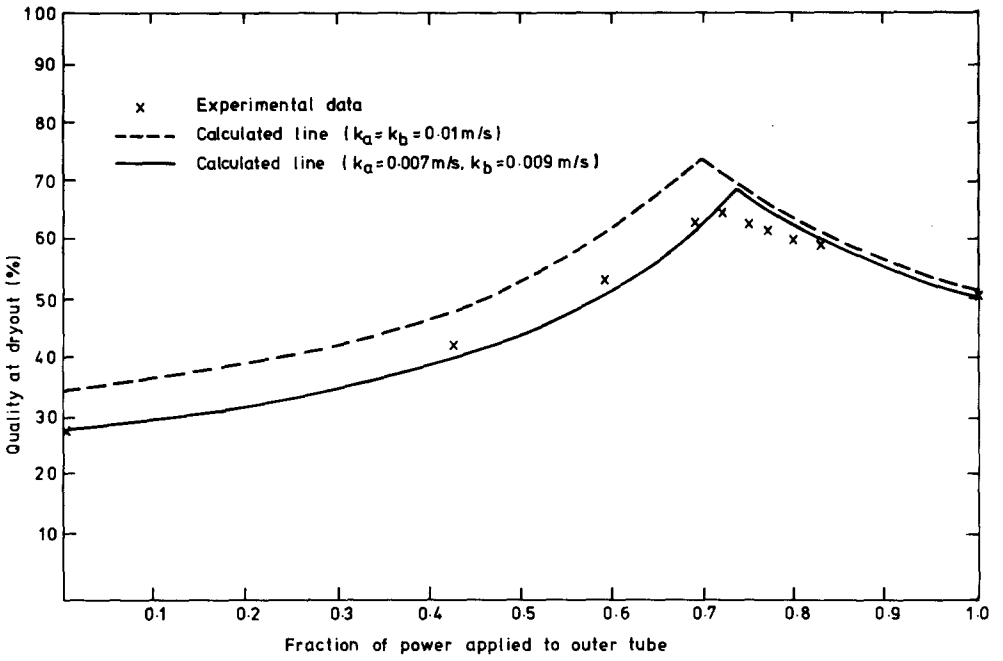

Figure 2. Comparison of the predictions of Whalley *et al.* (1975) with results of Jensen & Mannov (1974). (Taken from Whalley *et al.* (1975).)

greater than in a heated flow which has, say, the same quality at dryout as the quality of the unheated flow.

The most interesting result of the above analysis, namely that the mass transfer coefficient for deposition onto a surface depends mainly on the film thickness on that surface justifies in a qualitative way the method of subdivision into subchannels employed by Whalley *et al.* (1975) and Whalley (1976) in their analyses of flow through annuli and rod bundles.

Finally we note that the relationship given by [2.32] could be fed into a model for annular flow in an annulus and would remove the need for optimization of the predictions of the model.

NOMENCLATURE

- a inner radius of annulus
- b outer radius of annulus
- C concentration of droplets
- D deposition mass flux
- E entrainment mass flux
- H step function
- $I(r_2, r_1)$ defined by [2.16]
- J_0 Bessel function of first kind or order zero
- k mass transfer coefficient
- m film thickness
- $\mathbf{n}$ unit outward normal
- r radial coordinate
- r_1, r_2 inner and outer plug source radii
- $\hat{\mathbf{r}}$ unit vector in outward radial direction
- S source term
- s dummy subscript
- T solution term in diffusion equation, defined by [2.13], [2.14]

t transformed variable, defined by [2.5]
U mean gas core velocity
U_0 defined by [2.15]
V elementary volume
V^* typical gas core velocity
Y_0 Bessel function of second kind of order zero
z axial coordinate

Greek symbols

α_n root of $U_0(\alpha_n a) = 0$
δ Dirac delta function
ϵ distance of entrainment source from wall
λ diffusion coefficient
Σ elementary surface area
τ, τ^* time
χ arbitrary function of r

Subscripts

a inner surface of annulus
b outer surface of annulus
av average over annulus
0 origin for coordinate transform
$\bar{}$ an overbar denotes asymptotic ($t \to \infty$) limit
∇_r^2 radial Laplace operator
∇_r $\hat{\mathbf{r}}$ component of gradient operator

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